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Letter to the Editor

On Recursion Relations for Rapid Computation of Rotation Functions*

In a recent Note, Walker [1] has derived a recursion relation for computing a sequence of functions $d^J_{MK}(\beta)$ with M and K fixed, and J ranging upward from its minimum value. The primary purpose of this Letter is to draw attention to the classic work of Gelfand and Shapiro [2], on representations of the group of rotations of three-dimensional space and their applications, in which essentially an identical recursion already exists. Their relation is Eq. (9′) on p. 288 in which $P^l_{jm}(\mu)$ plays the role of $d^J_{MK}(\beta)$. The precise connection between these two functions is

$$P^J_{KM}(\beta) = (-1)^{(M-K)/2}\, d^J_{MK}(\beta), \tag{1}$$

where $\mu \equiv \cos\beta$. Also, Walker [3] has recently discovered an additional place in the literature [4] where the recursion exists.

Another useful recursion relation is available in which J and one of the subscripts in $d^J_{MK}(\beta)$ are fixed. The relation was first given in Ref. [2], in the formula following Eq. (6′) on p. 286. This recursion, in a modified form, has already been implemented by Fox and Ozier [5]. More recently, Fox and Krohn [6] have calculated all the values of $d^J_{KM}(\pi/2)$ for $0 \leqslant J \leqslant 100$ in 4.6 sec and 8.8 sec using single- and double-precision arithmetic, respectively, on a CDC 7600 computer. The values agreed to at least 11 significant figures in these two calculations.

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